AD-A033 515 SYRACUSE UNIV N Y DEPT OF INDUSTRIAL ENGINEERING AND--ETC F/G 12/2
RELIABILITY ACCEPTANCE SAMPLING PLANS BASED UPON PRIOR DISTRIBU--ETC(U)
SEP 76 A L GOEL, A M JOGLEKAR F30602-71-C-0312
UNCLASSIFIED TR-76-1 RADC-TR-76-294-VOL-1 NL

1 OF 1
AD
A033515

END
DATE
FILMED
2-77

12

RADC-TR-76-294, Volume I (of five)
Final Technical Report
September 1976

RELIABILITY ACCEPTANCE SAMPLING PLANS BASED UPON PRIOR DISTRIBUTION
Introduction and Problem Definition

Syracuse University

ROME AIR DEVELOPMENT CENTER
AIR FORCE SYSTEMS COMMAND
GRIFFISS AIR FORCE BASE, NEW YORK 13441

This report contains a large percentage of machine-produced copy which is not of the highest printing quality but because of economical consideration, it was determined in the best interest of the government that they be used in this publication.

This report has been reviewed by the RADC Information Office (OI) and is releasable to the National Technical Information Service (NTIS). At NTIS it will be releasable to the general public, including foreign nations.

This report has been reviewed and approved for publication.

APPROVED: Anthony Coppola

ANTHONY COPPOLA
Project Engineer

APPROVED: Joseph J. Naresky

JOSEPH J. NARESKY
Chief, Reliability & Compatibility Division

FOR THE COMMANDER: John P. Huss

JOHN P. HUSS
Acting Chief, Plans Office

Do not return this copy. Retain or destroy.

MISSION
of
Rome Air Development Center

RADC plans and conducts research, exploratory and advanced development programs in command, control, and communications (C^3) activities, and in the C^3 areas of information sciences and intelligence. The principal technical mission areas are communications, electromagnetic guidance and control, surveillance of ground and aerospace objects, intelligence data collection and handling, information system technology, ionospheric propagation, solid state sciences, microwave physics and electronic reliability, maintainability and compatibility.

AMERICAN REVOLUTION BICENTENNIAL 1776-1976

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE

19

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
18 RADC-TR-76-294-Vol I [redacted] 1

2. GOVT ACCESSION NO.

3. RECIPIENT'S CATALOG NUMBER
9

4. TITLE (and Subtitle)
6 RELIABILITY ACCEPTANCE SAMPLING PLANS BASED UPON PRIOR DISTRIBUTION. Volume I. Introduction and Problem Definition.

5. TYPE OF REPORT & PERIOD COVERED
Final Technical Report, June 1975 - August 1976,

6. PERFORMING ORG. REPORT NUMBER
14 TR-76-1

7. AUTHOR(s)
10 Amrit L./Goel[1]
Anand M./Joglekar[2]
(see reverse)

8. CONTRACT OR GRANT NUMBER(s)
15 F30602-71-C-0312

9. PERFORMING ORGANIZATION NAME AND ADDRESS
Syracuse University
Sky Top Office Building
Syracuse NY 13210

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
16 45400526 62702F
17 05

11. CONTROLLING OFFICE NAME AND ADDRESS
Rome Air Development Center (RBRT)
Griffiss AFB NY 13441

12. REPORT DATE
11 Sep[redacted] 1976

13. NUMBER OF PAGES
14

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)
Same
12 24p.

15. SECURITY CLASS. (of this report)
UNCLASSIFIED

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE
N/A

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)
Same

18. SUPPLEMENTARY NOTES
RADC Project Engineer:
Anthony Coppola (RBRT)

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Reliability	Bayesian Tests
Statistics	Statistical Tests
Testing	Bayesian Statistics
Reliability Tests	Prior Distributions

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

This report provides all the necessary information to make practical application of acceptance tests based on a prior distribution, when an exponential distribution of failures and an inverted gamma prior distribution are assumed. Volume I, "Introduction and Problem Definition," presents an introduction to the problem and a summary of the other volumes. Volume II, "Risk Criteria and Their Interpretation," defines the different statistical risks which can be used to characterize the tests and provide a guide for selecting appropriate risks in various practical situations. Volume III, "Implications and

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

409 184

LB

Determination of the Prior Distribution," provides the means for determining the parameters of the prior distribution from existing data, and discusses the reason for using an inverted gamma. Volume IV, "Design of Testing Plans," provides instructions for establishing a test time and number of allowable failures based on the prior distribution and the selected risks. Volume V, "Sensitivity Analyses," shows the effects on the test parameters caused by changes in the prior parameters.

Block 7 (Continued)

[1] Associate Professor, Department of Industrial Engineering and Operations Research; and Computers and Information Science, Syracuse University, Syracuse, New York 13210.

[2] Research Associate, Department of Industrial Engineering and Operations Research, Syracuse University, Syracuse, New York 13210. Now with The Pillsbury Company, Minneapolis Minnesota.

EVALUATION

1. This report describes the results of one task performed by Syracuse University under contract AF30602-71-C-0312. The objective of this task was to provide the necessary framework for practical application of reliability tests which make use of information available before the test begins. Such tests have been extensively discussed in the technical literature, since the use of available data would seem to make it possible to reduce test time without loss of statistical confidence. However, the discussions have produced a great deal of controversy which, in turn, created an understandable reluctance on the part of equipment developers to apply the procedures. Syracuse's task was to develop from the available literature a sound approach and all necessary procedures to permit the practical application of a reliability test using prior data. They fully met this objective.

2. The report is comprehensive, providing guidance in selecting appropriate test risks for different procurement situations, procedures for obtaining the required statistical parameters from the existing data, the means of determining the test length and acceptance criteria, and a sensitivity study showing the effects on the test of changes in the prior data. If one accepts the two basic assumptions that failures will follow an exponential distribution and that the prior distribution of equipment mean-time-between-failure is an inverted gamma, there can be no controversy over the use of the plans. Neither of these assumptions can be guaranteed to hold in all cases, but both are considered by this evaluator as quite reasonable. Hence, with the information in this report, test plans can be developed and applied without exceeding the bounds of reasonable prudence. The next step in that direction will be the creation of a document which presents the information in a form which can be easily used by a program manager to develop a test plan for his particular situation.

3. Though the required use of computer programs in formulating the test plans will impose some degree of difficulty on the user, RADC intends to provide an exhibit which will make it as easy as possible to generate test plans which take advantage of prior information.

Anthony Coppola
ANTHONY COPPOLA
Chief, R&M Engineering Techniques Section
Reliability Branch

PREFACE

This report is one of a set of five presenting the results of part of the work done under contract number F-30602-71-C-0312. The report is delivered to RADC in accordance with item A006 of the Contract Data Requirement List. Sponsorship and technical direction of this task originated in the Reliability and Maintainability Engineering Section (A. Coppola, Chief), Reliability Branch (D. Barber, Chief), within the Reliability and Compatibility Division (J. Naresky, Chief) of the Rome Air Development Center. Mr. Anthony Coppola was the Project Engineer who was technically supported by Mr. Jerome Klion.

The titles of the reports on the subject "Reliability Acceptance Sampling Plans Based Upon Prior Distribution" are as follows:

Volume I.	Introduction and Problem Definition.
Volume II.	Risk Criteria and Their Interpretation.
Volume III.	Implications and Determination of the Prior Distribution.
Volume IV.	Design of Testing Plans.
Volume V.	Sensitivity Analyses.

ABSTRACT

The problem of the design and analysis of reliability acceptance sampling plans in the presence of heterogeneous production is discussed in this report. A summary of the work done on various aspects of this problem is also presented.

ACKNOWLEDGMENTS

The authors are thankful to Anthony Coppola, Anthony Feduccia and Jerome Klion of RADC for their helpful suggestions. We are particularly indebted to Donald Wu for his valuable programming help and other assistance during the course of this project.

TABLE OF CONTENTS

1. INTRODUCTION AND PROBLEM DEFINITION

Reliability is the probability of a device performing its function adequately for the intended period of time. If the time to failure, t, has an exponential distribution given by

$$f(t|\theta) = \frac{1}{\theta} e^{-t/\theta} \quad ; \; t \geq 0, \; \theta > 0 \tag{1}$$

then the following measures of reliability may be interchangeably used.

Reliability, $R(t) = P(t > T) = e^{-T/\theta}$

Mean Time Between Failures (MTBF) = θ

Hazard Rate, $h(t) = 1/\theta$

(1-p)th Quantile of Failure Distribution, $t_p = -\theta \ell n p$

In the following we shall take θ as the measure of reliability.

Two types of life tests are generally performed to ensure that adequate reliability has been achieved. These are demonstration tests and acceptance sampling tests. Demonstration tests are performed to evaluate the lot MTBF with respect to the specified MTBF, θ_0. These tests may be conducted on initial production samples or after major design and development changes have been made. After production starts, acceptance sampling tests are performed to ensure continuing production of acceptable

equipments. We are primarily concerned with the design and analysis of acceptance sampling plans.

The situation in acceptance sampling is typically the following. Accept/reject decisions are to be made regarding a sequence of repairable systems or lots containing a large number of components. The decisions are based upon test results subject to random fluctuations. A loss is incurred when either an accept or a reject decision is wrongly taken. Further, decisions regarding each lot or system are taken independently of the results for other lots or systems.

In the classical context, acceptance sampling plans are designed to distinguish between two values of θ, namely the minimum acceptable MTBF θ_1 and the specified MTBF θ_o. The loss corresponding to wrong decisions is quantified by the producer's risk $\alpha = P(R|\theta_o)$ and the consumer's risk $\beta = P(A|\theta_1)$ where A and R denote acceptance and rejection respectively. In this context, various types of single and sequential sampling plans are employed to limit α and β to specified values. For example, a truncated single sample plan for a system is used as follows. The plan consists of placing a system on a life test of duration

T leading to an accept decision if the observed number of failures r is less than or equal to the acceptance number r*. The design of a sampling plan consists of obtaining the parameters T and r* such that the specified producer's and consumer's risks are satisfied.

The classical sampling plans have the disadvantage that they do not give due weight to the frequency with which different values of θ occur in practice and thereby lead to risk criteria which may not be very meaningful to the producer and/or the consumer. In practice, the variability casued by the nature of the production process is likely to be such that the values of θ differ from lot to lot. Under suitable assumptions, the variability in θ may be expressed by the frequency distribution for θ. This frequency distribution, $g(\theta)$ is usually called the prior distribution of θ. For meaningful results, due consideration should be given to $g(\theta)$ by incorporating it in the design of sampling plans.

The problem under consideration in this study, then, is the design and analysis of single sample reliability acceptance plans for exponential distribution when θ is a random variable with distribution $g(\theta)$. The four facets of the problem are:

(a) Interpretation and choice of appropriate risk criteria to meet producer's and consumer's true interests in various situations of practical interest.

(b) Determination of the functional form and parameters of the prior density.

(c) Design of single sample truncated, censored, with replacement and without replacement plans for systems and lots. The design is based upon exponential failure density, using the appropriate prior density and risk criteria.

(d) Investigation of the uncertainty associated with prior parameters and determination of the effect of this uncertainty on designed plans and risks.

A summary of the work done on these four aspects is given in Section 3.

2. LITERATURE REVIEW

Three somewhat related approaches have been considered in the literature toward the design of reliability acceptance sampling plans based upon prior knowledge regarding θ. The first uses the posterior distribution of θ and the accept/reject decision is made on the basis of Bayesian confidence interval for θ. The second approach is to explicitly define the producer's and consumer's risks using $g(\theta)$ and then to design the plans to satisfy these risks. The third approach converts $g(\theta)$ into pseudo number of failures and pseudo hours of test time. Amongst the three, the second approach is most general and meaningful due to its explicit consideration of risks.

One of the earliest efforts toward the solution of this problem using the first approach is due to Pinkham and St. James (1963). They considered a known two point prior for θ such that $P(\theta = \theta_0) = \pi$ and $P(\theta = \theta_1) = 1 - \pi$. They used a classical approach with $P(R|\theta_0) = \alpha$ and $P(A|\theta_1) = \beta$ such that the probability of correct identification is equal to $\pi(1-\alpha) + (1-\pi)(1-\beta)$. The risks α and β were chosen to meet a specified probability of correct identification and the plans were designed to satisfy these risks. Bonis (1966) used the posterior distribution of θ to make the accept/reject decision. The criterion

was to accept if $P(\theta = \theta_0|\text{data}) = 1 - \alpha$. It should be noted that these two procedures do not adequately take into consideration the producer's and the consumer's risks.

Following the second approach, the design of fixed time reliability sampling plans based upon the producer's and consumer's risks has been considered by Balaban (1969), Blumenthal (1973), Easterling (1970), Schick and Drnas (1972) and Schafer (1973). The general approach taken in these papers is to treat the time to failure as having an exponential distribution and to use the conjugate prior, inverted gamma, for the parameter, θ. The producer's and the consumer's risks are defined and these lead to two simultaneous equations in test time T and acceptance number r*. The test plan is obtained by computing (T,r*) from the two equations. The design criteria proposed by these authors, in our notations, are given below.

Balaban (1969):

Producer's Risk: $P(R|\theta = \theta_0) = \alpha$

Consumer's Risk: $P(\theta \leq \theta_1|A) = \beta^*$

Easterling (1970):

Producer's Risk: $P(R|\theta \geq \theta_0) = \bar{\alpha}$

Consumer's Risk: $P(A|\theta \leq \theta_1) = \bar{\beta}$

Schick and Drnas (1972):

Producer's Risk: $P(\theta \geq \theta_0 | R) = \alpha^*$

Consumer's Risk: $P(\theta \leq \theta_1 | A) = \beta^*$

Schafer (1971 and 1973):

Producer's Risks: $P(R) = 1 - P(A)$; $P(\theta \geq \theta_0 | R) = \alpha^*$

Consumer's Risk: $P(\theta \leq \theta_1 | A) = \beta^*$

As indicated earlier, the third approach is to convert the information contained in the prior distribution $g(\theta)$ into equivalent test hours δ and equivalent number of failures ϕ. An appropriate test is designed to obtain T and r* to satisfy the risks α and β. The actual test procedure is to conduct the test for $(T-\delta)$ hours and accept if the number of failures is less than or equal to $(r^*-\phi)$. Such plans are considered by Feduccia (1970), and by Guild (1973).

Schafer and Singpurwalla (1969), Barnett (1972), and MacFarland (1971) have considered the design of sequential reliability demonstration plans using prior distribution. The criteria suggested by Schafer et al and Barnett are identical. For the case of exponential failure distribution and inverted gamma prior density, the criterion is to accept at the n th stage of sequential experimentation if

$P(\theta \geq \theta_1 | \hat{\theta}_n) \geq k_1$ and reject if $P(\theta \geq \theta_1 | \hat{\theta}_n) \leq k_2$ where $\hat{\theta}_n$ is the estimate of θ obtained from the failure data at the n th stage and k_1 and k_2 are two prespecified real numbers.

MacFarland (1971) considered the case of two points (discrete) prior and used the ratio of posterior probabilities for making the decision. The criterion is, at the n th stage

accept if

$$\frac{P(\theta = \theta_0 | \hat{\theta}_n)}{P(\theta = \theta_1 | \hat{\theta}_n)} \leq \frac{\beta}{1-\alpha},$$

and reject if

$$\frac{P(\theta = \theta_0 | \hat{\theta}_n)}{P(\theta = \theta_1 | \hat{\theta}_n)} \geq \frac{1-\beta}{\alpha}.$$

The work of Schafer et al (1970, 1971, 1973) is an extensive investigation of the design of reliability sampling plans for exponential distribution based upon a prior distribution for θ. The three reports labelled Phase I, Phase II and Phase III are now reviewed in some detail. Phase I primarily deals with the fitting of a prior distribution, the objective being to ascertain if inverted gamma is a suitable prior to consider. Also, the requirements for the data and moment estimators

for fitting prior distribution are studied. On the basis of failure data obtained on different equipments they show that the inverted gamma is a reasonable prior to consider. Phase II contains a brief overview of the available design criteria. On the basis of a limited study the authors conclude that (α^*, β^*) are the proper criteria to use. The report also contains some discussion of test criteria when there is a lack of agreement between the producer and the consumer regarding the prior distribution. The methods of modifving the prior distribution in view of additional data or due to changes in environment are also considered. Phase III recommends a different set of risk criteria, namely P(A) and β^*, where P(A) is the unconditional probability of acceptance. Single sample fixed time tests are tabulated for two sets of criteria $\{P(A), \beta^*\}$ and $\{P(A), \beta\}$. Also, plans are given for the sequential approach due to Schafer and Singpurwalla (1970). Some methods of updating the prior distribution and some tests to detect shifts in the prior are also given in Phase III.

3. SUMMARY OF REPORTS II TO V

A summary of the work reported in the technical reports entitled "Reliability Acceptance Sampling Plans Based Upon Prior Distribution-Part II - Risk Criteria and Their Interpretation, Part III - Implications and Determination of the Prior Distribution, Part IV - Design of Testing Plans, and Part V - Sensitivity Analyses," is given below.

Risk Criteria and Their Interpretation (Part II)

The definitions and the mathematical, physical and graphical interpretations of the following risk criteria are given.

1. The probability $P(A|\theta = \theta_1)$ that a system (lot) with minimum acceptable MTBF is accepted.
2. The probability $P(R|\theta = \theta_0)$ that a system with specified MTBF is rejected.
3. The probability $P(A|\theta \leq \theta_1)$ that a system which is of unacceptable reliability is accepted.
4. The probability $P(R|\theta \geq \theta_0)$ that a system of acceptable reliability is rejected.
5. The probability $P(\theta \leq \theta_1|A)$ that the MTBF of a system which has been accepted is less than the minimum acceptable MTBF.

6. The probability $P(\theta \leq \theta_0|A)$ that the MTBF of a system which has been accepted is less than the specified MTBF.

7. The probability $P(\theta \geq \theta_0|R)$ that the MTBF of a system which has been rejected is greater than the specified MTBF.

8. The unconditional probability $P(R)$ that a randomly selected system is rejected.

The interrelationships between the risks are explored and the exact protection provided to the producer and the consumer by the use of these risks is determined. The choice of risk combinations that best reflect the primary interests of the producer and the consumer in various situations of practical interest is discussed at length.

Implications and Determination of the Prior Distribution (Part III):

This report provides a detailed discussion of the implications of the existence of a prior distribution in the context of reliability acceptance sampling. Various situations under which such distributions can arise are described and their implications explored. The choice of the functional form of the prior distribution and the estimation of parameters of this distribution for various types of reliability data are discussed. Specifically, the prior for the exponential parameter θ is chosen to be the two parameter inverted gamma and is shown to be

adequate for some observed failure data. Maximum likelihood estimation of prior parameters is considered when the failure data arise from truncated, censored, with replacement and without replacement single sample plans for systems and lots. A comprehensive example is included to illustrate the estimation of prior parameters and their variance-covariance matrix when data on the number of failures in fixed time is available.

Design of Testing Plans (Part IV)

This report contains numerical and graphical procedures for the design of single sample truncated plans for systems and lots. The graphical procedure has the advantage that it permits the exploration of design region to choose between competing designs under various practical constraints. Several examples are given to illustrate the usefulness of the graphical method. The numerical procedure can be used when the contour nomograms for the graphical procedure are not available and/or an exploration of the design region is not considered necessary.

Sensitivity Analysis (Part V)

In this report we examine the effects of the uncertainties associated with the prior parameters on the designed plans and the selected risks. The uncertainty associated with the parameter estimates is quantified by obtaining the joint confidence regions for the two parameters via simulation. The likelihood plots and contours of the asymptotic bivariate normal density are also examined in an effort to assess the effect of the uncertainty on the designed plans and the risks.

4. REFERENCES

1. Balaban, H. (1969) 'A Bayesian Approach to Reliability Demonstration', Annals of Assurance Sciences, Vol. 8, No. 2, pp. 497-506.

2. Barnett, V.D. (1972) 'A Bayesian Sequential Life Test' Technometrics, Vol. 14, No. 2, pp. 453-468.

3. Blumenthal, S. (1973) 'Reliability Demonstration', TR No. 183, Dept. of Operations Research, Cornell University.

4. Bonis, A.J. (1966) 'Bayesian Reliability Demonstration Plans', Annals of Reliability and Maintainability, Vol. 5, pp. 861-873.

5. Easterling, R.G. (1970) 'On the Use of Prior Distributions in Acceptance Sampling', Annals of Reliability and Maintainability, Vol. 9, pp. 31-35.

6. Easterling, R.G. (1972) 'A Personal View of Bayesian Controversy in Reliability and Statistics', IEEE Transactions on Reliability, Vol. R-21, No. 3, pp. 186-194.

7. Feduccia, A.J. (1970) 'A Bayesian/Classical Approach to Reliability Demonstration', RADC-TR-70-72, (871969).

8. Feduccia, A.J. (1973) 'A Comparison of Classical and Bayesian Reliability Demonstration Tests', M.S. Thesis, Syracuse University, Syracuse, New York.

9. Guild, R.D. (1973) 'Bayesian MFR Life Test Sampling Plans', Journal of Quality Technology, Vol. 5, No. 1, pp. 11-15.

10. MacFarland, W.J. (1971) 'Sequential Analysis and Bayesian Demonstration', Annals of Reliability and Maintainability, Vol. 10, pp. 24-38.

11. Pinkham, R.S. and L.N. St. James (1963) 'An Approach to System Reliability Verification by Test' Proceedings Second Annual Aerospace Conference on Reliability and Maintainability, Vol.10, pp. 24-38.

12. Schafer, R.E. (1964) 'Bayesian Operating Characteristic and Quality', Industrial Quality Control, pp. 118-122.

13. Schafer, R.E. and Singpurwalla, N.D. (1970) 'A Sequential Bayes Procedure for Reliability Demonstration' Naval Research Logistic Quarterly, Vol. 17, pp. 55-67.

14. Schafer, R.E. et al (1970) 'Bayesian Reliability Demonstration: Phase I - Data for the A-Priori Distribution, RADC-TR-69-381, (863199).

15. Schafer, R.E. et al (1971) 'Bayesian Reliability Demonstration: Phase II - Development of A-Priori Distribution', RADC-TR-71-209, (732283).

16. Schafer, R.E. et al (1973) 'Bayesian Reliability Demonstration: Phase III - Development of Test Plans', RADC-TR-73-139, (765172).

17. Schick, G.J. and T.M. Drnas (1972) 'Bayesian Reliability Demonstration', AIIE Transactions, Vol. 4, No. 2, pp. 92-102.